Addendum to "Large Integral Points on Elliptic Curves" (Math. Comp., v. 48, 1987, pp. 425–436)

By Don Zagier

The author has been informed by Serge Lang that the conjecture that all integral solutions of $y^2 = x^3 + ax + b$ are polynomially bounded in a and b (cf. [3, Section 2]), and the observation that a naive probabilistic argument would lead to the expectation that the number $\rho = (\log x)/\log(\max(|a|^{1/2}, |b|^{1/3}))$ does not ever exceed 10 + o(1), have been made by him and by Harold Stark; Lang conjectured that all solutions, with the possible exception of a finite number of parametric families, satisfy $\rho \leq 10 + o(1)$ (the necessity of the caveat about exceptional families followed from a related example of Stark's). For all this, see [1]. It has been shown by Paul Vojta that Lang's conjecture in this form follows from his own more general Diophantine conjecture [2]. Recently, Noam Elkies has constructed an infinite family of curves having an integral solution with $\rho = 12 + o(1)$.

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1. S. LANG, "Conjectured Diophantine estimates on elliptic curves," in Arithmetic and Geometry (M. Artin and J. Tate, eds.), Vol. I, Birkhäuser, Boston, 1983, pp. 155–171.

2. P. VOJTA, Diophantine Approximations and Value Distribution Theory, Lecture Notes in Math., vol. 1239, Springer, Berlin, 1987.

3. D. ZAGIER, "Large integral points on elliptic curves," Math. Comp., v. 48, 1987, pp. 425-436.

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